The pre-stressed piezoelectric sandwich transducer

E.A. NEPPIRAS, D.Sc., A.R.C.S., F.Inst.P.

The pre-stressed sandwich transducer construction is now preferred for many low-frequency ultrasonic applications, notably in the field of sonar and macrosonics. This paper reviews historical background and discusses the factors involved in the rational design of transducers of this type. Modern designs of asymmetric and complex multi-element sandwiches are dealt with in detail, with emphasis on the practical problems involved in their construction

Introduction

The simplest form of pre-stressed sandwich transducer is a disc, or paired-discs, of piezoelectric ceramic sandwiched between metal end-sections and placed under compression-bias by means of bolts drawing up on the metals (Fig. 1). The transducer is a $\lambda/2$ resonator and is restricted to the extensional-mode of vibration. At present approximately 80% of all low-frequency extensional-mode transducers are of this type.

The origin of the sandwich construction can be traced back to the very first technological application of ultrasonics, Langevin's development of underwater signalling (1917) [1]. In those early days, quartz crystal was the only useful transducer material available and Langevin was faced with the problem of designing a low-frequency resonator using only thin quartz plates as the active material. He solved it by sandwiching the quartz between steel end-sections, thereby lowering the frequency and economizing in quartz. This construction was widely used in underwater applications, but not in the early experiments in macrosonics, probably because of its inherent mechanical weakness.

When piezoelectric ceramics first came into use for low-frequency applications (1964) [2], at first the transducers were in the form of simple rectangular blocks or tubes. Unlike quartz, barium titanate is cheap enough to use in this way. However, this simple arrangement is inherently weak. The tensile strength of the ceramic is low, and, just as important, it cannot be bonded strongly to the metal stubs or plates needed for coupling into the load. Because of these limitations, the Langevin sandwich design was reinvestigated, and by the simple process of pre-stressing, both of these difficulties were removed at one blow.

In the simplest pre-stressed sandwich, using a single crystal plate, one of the metal end-sections is at high electrical

potential and the pre-compressive bolt must be taken through an insulating washer so as not to make contact with both end-sections. This is very incovnenient and limits the amount of compressive force that can be used. In the accepted improved version two crystal plates are used, poled in opposite directions. The centre electrode is live and both end-sections are grounded (Fig. 1).

Historically, the first mention of a pre-stressed sandwich design is a patent dated 1955, with priority 1952 [3]. This describes an accelerometer, that is, a receiver-type device. H.B. Miller [4, 5] claims to be the first to outline a pre-stressed sandwich design for emitter applications. However, there is little doubt that these designs were also being explored and used in other sonar research centres about this time [6-9].

The advantages of the basic design of pre-stressed sandwich, illustrated in Fig. 1 can be summarized as follows:

- Material-costs are low as only thin discs of ceramic are needed.
- For the same reason, the crystal capacity is high and its electrical impedance low. This is a positive advantage at the low frequencies these transducers are operated at.
- 3) The pre-compression increases the mechanical strength.
- 4) The metal end-sections are good heat-sinks, well-coupled to the ceramic, so that the transducer can be driven at higher levels than would otherwise be possible.
- The metal end-sections permit a metal-to-metal bond to plates and stubs. These advantages explain why the design is so popular.

As far as the electrical, mechanical and electro-mechanical design of a sandwich transducer is concerned, the simple basic construction of Fig. 1 loses no generality if the single

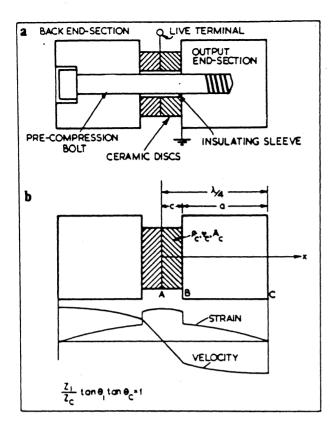


Fig. 1 The basic form of symmetrical pre-stressed sandwich transducer showing (a) construction and (b) distribution of velocity and strain.

ceramic disc is replaced by several symmetrically-placed discs, or pairs of discs, thereby avoiding the need to cut a hole in the centre of the transducer plate [7]. It also includes the case where the single central bolt is replaced by several peripheral bolts [8]. These latter designs avoid putting a hole through either the ceramic or the metal end-sections. Peripheral bolting is claimed to give more uniform stressing of the transducer discs and the design is efficient for very high power operation [3, 10].

The basic sandwich construction

The frequency equation

Consider the basic form of sandwich illustrated in Fig. 1. It is assumed that the transducer is resonant in the free-free $\lambda/2$ -extensional-mode and that the nodal plane divides the ceramic equally. Each $\lambda/4$ section can then be dealt with separately. By considering the propagation of a plane stress wave through the two materials in the x-direction, applying resonant boundary-conditions at the two ends A,C and velocity- and force-continuity at the discontinuity B, the frequency-determining relation is obtained:

$$\tan \theta_c \tan \theta_1 = Z_c/Z_1 = R \tag{1}$$

where

$$- \theta_c = \omega c/v_c \quad \theta_1 = \omega a/v_1 \quad Z_c = \rho_c v_c A_c; \ Z_1 = \rho_1 v_1 A_1$$

and v is the extensional-mode velocity of sound in the material. To a rough first approximation, the end-sections move as lumped masses on the ceramic plates, which act as springs. Fig. 1 includes velocity and strain distributions in a

typical transducer of this sort. Equation 1 relates the resonance frequency ω to the two distances a and c and the impedance-ratio R. It is, therefore, clear that many different shapes and material-combinations will give the same resonance frequency. Some scope, therefore, exists for designing for other requirements. For example, the power-handling capacity or electrical impedance could be modified by choosing a suitable disc-thickness.

After choosing the materials, the resonance frequency and any two of the three geometrical quanties a, c and A

 $\frac{A_c}{A_1}$, Equation 1 can be used to fix the unknown dimension.

Based on this equation, universal design curves can be constructed relating the phase-lengths of the two materials over a range of impedance-ratios. These are reproduced in Fig. 2. For a given transducer geometry, Equation 1 cannot be used directly to derive the resonance frequency as the latter cannot be expressed explicitly. However, the design curves of Fig. 2 can be used. It is only necessary to draw a straight

Fig. 2 can be used. It is only necessary to draw a straight line from the origin of slope $\frac{av_c}{cv_1}$. ω can then be obtained

from either co-ordinate at the point of intersection of this line with the curve of appropriate R-value.

Equivalent circuits

The above discussion refers to an unloaded free-free resonator. If the transducer operates into a complex acoustic load, the resonance frequency will change due to the change of boundary condition at the work-face. If the load is known, the new frequency equation can be derived as before. In practice, better insight is obtained by using an equivalent-circuit representation. The general equivalent circuit for the simple sandwich-type transducer is the 6-terminal equivalent for a crystal-plate [11] with added mechanical elements to represent the end-masses and pre-compression bolt. These are represented in the equivalent circuit as transmission lines. The effect of the bolt is often small and if it is omitted for the moment and it is assumed that the cera-

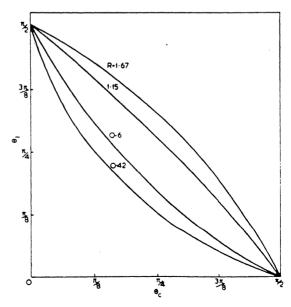
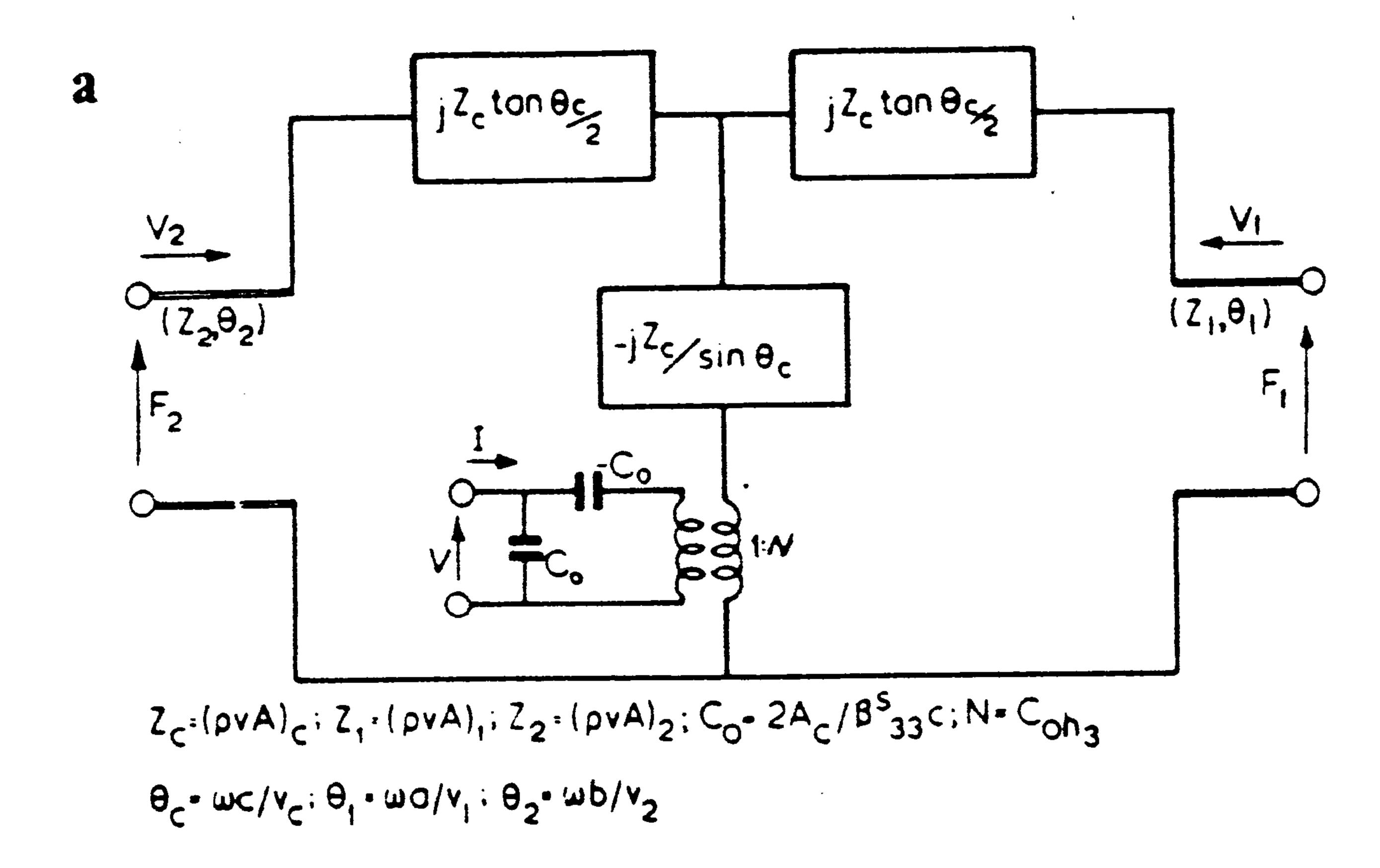


Fig. 2 Relation between the phase-lengths of the two portions of a $\lambda/4$ section of a sandwich transducer for various useful values of the impedance-ratio.



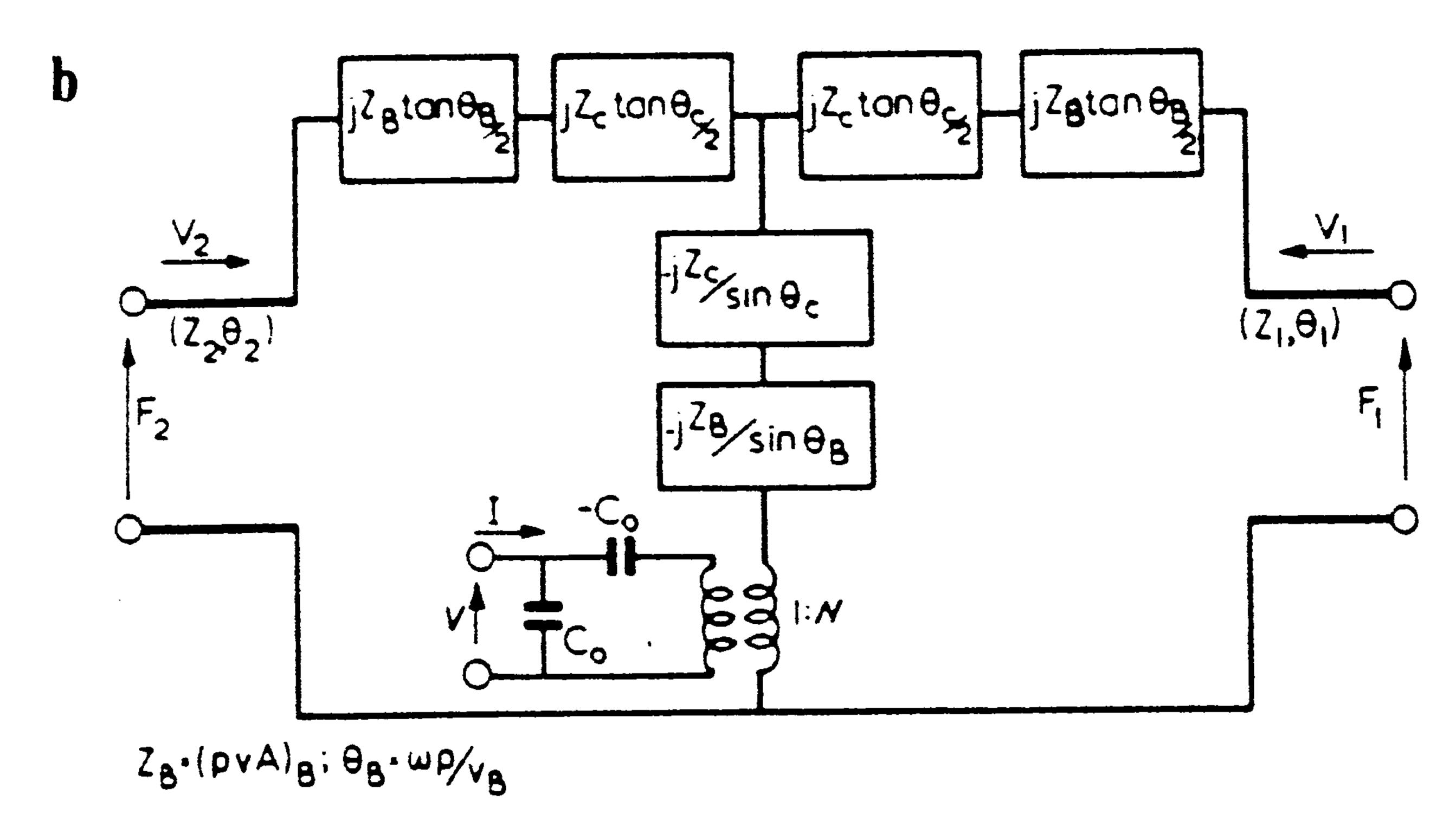


Fig. 3 Equivalent circuits for a pre-stressed sandwich transducer (a) approximate 6-terminal equivalent circuit, and (b) 6-terminal equivalent circuit including mechanical elements representing the pre-compression bolt.

mic and end-sections are untapered cylindrical elements, the circuit of Fig. 3a is an adequate representation of the system. In this circuit generalizations have been made by assuming that the end-sections are of different materials and, therefore, different impedances and lengths. For the simple sandwich of Fig. 1 θ_1 was put = θ_2 . This circuit also assumes that the elements are loss-free. However, 'linear' losses can be included formally by writing the elastic constants and the permittivity as complex numbers. The imaginary parts then represent the damping, through the appropriate Q-values. Where two crystal plates are used, as in Fig. 1, the clamped capacitance C_0 is, of course, twice the capacitance of one plate.

When the transducer is used as an emitter with one face loaded, the acoustic load, Z_L , is placed across one pair of mechanical terminals with the others short-circuited. The circuit then reduces to a two-terminal equivalent. The behaviour of the transducer can be examined if Z_L is known. As usual, for frequencies near resonance, the circuit elements can be 'lumped', resulting in a simpler circuit.

The pre-stressing bolt will add some stiffness and mass to the vibrator. However, its stiffness is usually much less than that of the ceramic and its mass much less than the end-sections. The circuit of Fig. 3a is, therefore, still a fair approximation, even for the pre-stressed transducer. However, if necessary, the circuit can easily be extended to include the dynamic effect of the bolt, as indicated in Fig. 3b. This circuit is still only an approximation, however, as the bolt has been idealized to a solid cylinder of phase length $2\theta R$ where

 $\theta_B = \frac{l\omega}{v_B}$, *l* being half the bolt length and v_B is extensional mode wave velocity. In all this, it must be remembered that the static pre-compression will modify the constants of the ceramic and it is these values that must be used in any design calculations.

Efficiency

Whether the transducer is used as an emitter or receiver, its performance can be expressed in terms of the following three parameters: effective electro-mechanical coupling (k_{eff}); unloaded mechanical Q-value (Q_m^D); and the dielectric Q-value (Q_e^S). The quantity $\psi = (k_{eff}^2 Q_m^D Q_e^S)$ is a useful figure-of-merit for transducers used as emitters, as the potential efficiency is an increasing function of this parameter, and approximately equal to $(1-2\psi^{-1/2})$ for reasonably-efficient transducers [12]. From this it can be seen that ψ must be increased above a few hundred before the efficiency becomes really high: The three quantities k_{eff}^2 , Q_m^D and Q_e^S are equally important in fixing the potential efficiency. They will now be considered separately:

1) k_{eff}^2 is defined as the ratio of converted stored energy to input stored energy. For all extensional-mode transducers k_{eff} is less than the material coupling (k_{33}) which is the relevant material constant quoted in tables. This is because not all of the dielectric energy is coupled to all of the elastic energy in these designs. k_{eff} will, therefore, depend on the phase-lengths and impedances of the various sections, and can be calculated as explained in Reference 13. For the simple symmetrical form of sandwich, as in Fig. 1, the following is obtained.

$$\left(\frac{k_{\text{eff}}}{k_{33}}\right)^{2} = \frac{8Z_{c}/\theta_{c}}{(2\theta_{1} - \sin 2\theta_{1})Z_{1} \sec^{2}\theta_{1}} + (2\theta_{c} + \sin 2\theta_{c})Z_{c} \csc^{2}\theta_{c}} \tag{2}$$

Fig. 4 has been plotted from Equation 2 and shows the effective coupling as a function of the phase-length of the ceramic for various impedance-ratios. It can be seen that $k_{\rm eff}$ reaches a peak value $(k_{\rm eff})_{\rm max}$ which approaches k_{33} for large values of the impedance-ratio. However, even for Z_c/Z_1 near unity $(k_{\rm eff})_{\rm max}$ need not be much lower than

 $\left(\frac{2\sqrt{2}}{\pi}\right)k_{33}$ which is the value obtained for a simple block transducer without end-sections. Below the maximum, the coupling falls off rapidly as the thickness of the ceramic is reduced and there is a practical minimum to the phase-length θ_c below which it is not advisable togo. For $\theta_c = \pi/10$ (or $c = \lambda_c/20$), k_{eff} is reduced to about $k_{33}/2$ for useful materials and these are convenient minimum values of θ_c or c to specify.

The unloaded mechanical Q-value is given by $Q_m^D = \omega_0^D M^*/R^* = S^*/\omega_0^D R^*$ where M^* , S^* , and R^* are the equivalent lumped mass, stiffness and mechanical resistance. For example, the equivalent mass is defined as that mass which when vibrating with the face velocity has the same kinetic energy as the whole vibrator.

Evaluating this for the basic sandwich of Fig. 1 gives,

$$\omega M^* = \frac{|Z_1|}{2} (2\theta_1 + \sin 2\theta_1)$$

$$+ \frac{Z_c \cos \theta_1}{2 \sin \theta_c} (2\theta_c - \sin 2\theta_c)$$
(3)

If the ceramic is thin and, therefore, contributes little mass, then putting $\theta_C = 0$ and $\theta_1 = \pi/2$ in Equation 3 gives $\omega M^* = \pi Z_1/2$ which, of course, is the expression for the equivalent mass of a straight cylindrical bar.

The mechanical resistance, R^* , is made up from: internal material-loss; losses at the contacting surfaces; and frictional damping in the screw-threads. The last-named is by far the most important. It cannot readily be calculated, but the overall mechanical loss resistance is easily measured, and this is the course adopted in practice.

3) The dielectric Q-value, Q_{ϵ}^{s} (= δ_{ϵ}^{-1}) is obtained from the tabulated or measured dielectric loss angle, δ_{ϵ} . This increases sharply with the excitation level. For the lead titanate zirconate ceramics used in modern emitter sandwiches δ_{ϵ} is typically in the range 0.005 - 0.0025 at very low drive. At high drive levels it may rise as high as 0.02 through increased dielectric hysteresis loss [14].

As a numerical example, for a typical well-constructed sandwich transducer, as used in ultrasonic cleaners,

$$k_{\rm eff}^2$$
 = 0.15; Q_m^D = 150; and Q_{ϵ}^s = 100, giving ψ = 2250 and $\eta_{\rm pot} \approx 95.7\%$.

Transforming sandwiches

In principle, if the potential efficiency is to be reached, the transducer should be acoustically matched into its load, that is to say, the impedance seen by the transducer should be near its optimum load impedance. In many applications, also, greater oscillatory amplitudes are needed than can be obtained from the transducer alone. Both these requirements can be met by coupling to the load through a resonant mechanical transformer[15]. However, it has been found that the higher the potential efficiency of the transducer, the less important impedance-matching becomes [12] and for typical sandwiches with efficiencies in the range 85%-97% there is little need to pay attention to matching. Still, it is often an advantage to spread the load by increasing the work-face area compared with that of the ceramic. The transducer must then be driven harder to reach the required work-face intensity, thereby making better use of the high power-handling capacity of the ceramic, but not necessarily increasing the electro-acoustic efficiency. This is most useful when operating into a liquid load, where it may be necessary to limit the work-face intensity to avoid cavitation (as in underwater signalling) or restrict its intensity (as in ultrasonic cleaning). The operating efficiency can always be measured in terms of the unloaded and loaded Q-values and compared with the potential efficiency.

With the sandwich construction, it is clearly possible to increase the work-face area by using a tapered-out metal stub for the output section. The design of such a structure is not difficult if the taper is of some recognized simple form, such as exponential, conical or catenoidal [16]. In some sonar designs, the tapered-out end-face is made so short that it can be regarded as a lumped mass, greatly simplifying the

design [6].

Equation 1 refers to only one $\lambda/4$ section of the $\lambda/2$ resonator. Clearly, any two $\lambda/4$ designs satisfying this equation may be bolted together to form a complete $\lambda/2$ resonator. The transducer will still be symmetrical if the nodal plane divides the ceramic equally. In general, the vibrator will then be a mechanical transformer, the two end-faces moving with different velocities. When loaded, the transducer will operate at different Q-values depending on which face is presented to the load. For the end-sections, metals of widely different ρ v's but the same area, are commonly used. By considering the propagation of a plane stress wave down the transducer, the distribution of velocity and strain is readily obtained as in Fig. 5. The following expression is obtained for the velocity transformation ratio [13]:

$$M = \left(\frac{\cos \theta_1}{\cos \theta_2}\right) = \left(\frac{1 + (Z_c/Z_2 \tan \theta_c)^2}{1 + (Z_c/Z_1 \tan \theta_c)^2}\right)^{\frac{1}{2}}$$
(4)

Clearly this transducer design is analogous to the well-known stepped-cylindrical velocity transformer. In fact, if the ceramic is very thin $(\theta_C \rightarrow 0)$ the total impedance ratio

 $\frac{Z_1}{Z_2}$ is a good approximation to the velocity ratio. The

'Shape-factor'* is then unity. The transducer has no value as a practical velocity transformer. If transformation were needed, an additional resonant stub would be used.

Transducers for liquid-load applications often use metal endsections for widely-different impedances, for example, steel and aluminium, where the impedance ratio is about 3. The greater bandwidth is obtained when the low-impedance aluminium faces the load. For example, a simple calculation shows that, for a resistive water load, the band-width is increased by the factor 4/3 for a steel-zirconate-aluminium sandwich compared with aluminium-zirconate-aluminium. For these transforming sandwiches $k_{\rm eff}$ and M^* can be calculated as for the simpler designs. The expressions are complex and are not quoted here. Instead, the reader is referred to Reference 13.

The power-handling capacity of sandwich transducers is limited by the mechanical strength of the ceramic. Under pre-stressing, the effective strength is increased from the normal ultimate-tensile, T, to(T+S) where S is the static compressive stress. Substituting for T_m in the relation $v_m/T_m = \phi/Z_1 = 1/Z_1$ where Z_1 is the characteristic impedance of the output section, an expression for the maximum allowable particle velocity is obtained: $v_m = (T+S)/Z_1$. The maximum intensity that can be transmitted into a resistive load of characteristic impedance Z_L is then $I_{max} = \frac{1}{2} Z_L v_m^2 = \frac{1}{2} Z_L \left[(T+S)/2Z_1 \right]^2$.

It is instructive to compare the particle velocity obtained at the end-face of a $\lambda/4$ section of a sandwich transducer with that which would be obtained from a block of the ceramic material resonant at the same frequency and driven at the same peak stress. Fig. 6 displays the square of this velocity-ratio, that is, the intensity-gain, as a function of the phaselength of the ceramic for various useful values of the impedance-ratio.

Asymmetric sandwiches

In many applications it is necessary to support the vibrator with the minimum of damping. A separate $\lambda/2$ resonant section may be added for this purpose, but usually the most

^{*} The 'Shape-factor' is a figure-of-merit for high-gain transforming devices. It is defined by ϕ where $v_m/T_m = \phi/Z$, v_m and T_m being the maximum velocity and stress developed in the device and Z the characteristic impedance of the material [15].

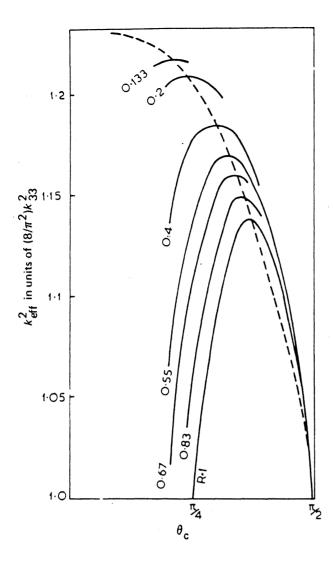


Fig. 4 Effective electro-mechanical coupling as a function of the phase-length of the ceramic material for a range of impedance-ratios.

convenient answer is to include a disc-type mounting flange in the sandwich assembly, as indicated in Fig. 7a. As the flange must be nodal and grounded, the ceramic must be moved away from the displacement node. The output portion is then a cylindrical $\lambda/4$ section, of length $\pi v_1/2\omega$ and the whole of the crystal plate is included in the other $\lambda/4$ section, the design of which is, of course, controlled by Equation 1.

In the general case of the asymmetric sandwich, with the ceramic moved away from the strain anti-node, it is easy to see that $k_{\rm eff}$ will be reduced, and Q_m^D increased, as compared with the basic symmetrical design. $k_{\rm eff}$ drops because the coupling between the dielectric and elastic strain energies is weakened; and Q_m^D increases because the losses at the contacting surfaces and in the ceramic are reduced. If these were the only mechanical losses involved, it is easy to see that shifting the ceramic should have no effect on the potential efficiency, as the parameter $k_{\rm eff}^2 Q_m^D$, and, therefore, $k_{\rm eff}^2 Q_m^D Q_{\epsilon}^S$, will remain unaltered by the move. However,

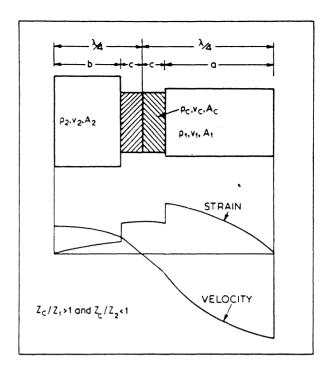


Fig. 5 Distribution of velocity and strain in a sandwich transducer with end-sections of different materials.

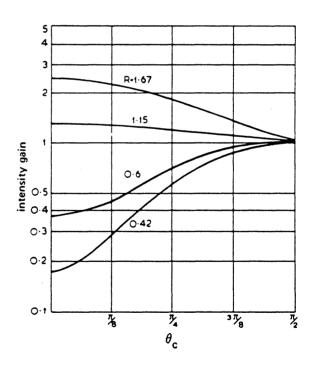


Fig. 6 Intensity-gain as a function of the phase-length of the ceramic portion of a sandwich transducer for various useful values of the impedance-ratio.

most of the mechanical loss occurs in the screw-threads and this is not reduced by moving the discs. The potential efficiency can, therefore, only be reduced by moving the ceramic. However, there is another factor to consider. If the transducer is assembled dry, perfect mechanical coup-

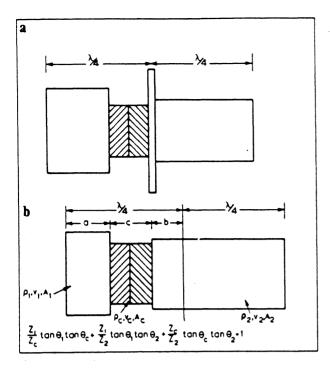
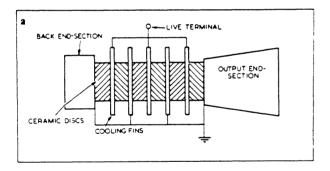


Fig. 7 Asymmetric sandwich transducers: (a) transducer with mounting flange; and (b) general form of simple asymmetric sandwich.



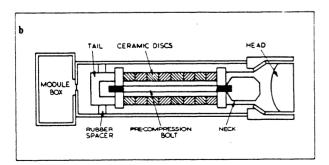


Fig. 8 Multi-element sandwich transducers; (a) transducer with cooling fins between ceramic discs; and (b) essentials of a typical pre-stressed sandwich as used in modern sonar.

ling between elements is only possible if the tolerance on flatness between contacting surfaces is less than the peak-to-peak oscillatory amplitude there. Some relaxation of the flatness requirements is, therefore, permissible if the discs are moved. This may be an important consideration in practice.

The input $\lambda/4$ section of the simplest asymmetric sandwich, shown in Fig. 7b, consists of three separate elements. The frequency equation can be derived in the same way as before, with the result given below, where the symbols are defined in the figure.

$$\frac{Z_1}{Z_c} \tan \theta_c \tan \theta_1 + \frac{Z_1}{Z_2} \tan \theta_2 \tan \theta_1
+ \frac{Z_c}{Z_2} \tan \theta_2 \tan \theta_c = 1$$
(5)

Multi-element sandwiches

Simple sandwich transducers have only one pair of ceramic discs, the minimum number consistent with having both endsections grounded. But clearly, any even number of stackedup discs could be used. One advantage of a multi-element sandwich of this sort is that a wide range of electrical impedances can be got by connecting the discs in alternative seriesparallel arrangements. Another is that power-handling capacity and coupling are both increased by increasing the number of discs. A disadvantage is that multiplying the number of contacting surfaces means that greater preparation and care are needed in the assembly. Overheating may also become a problem and to assist cooling, metal fins may be included between adjacent discs, (see Fig. 8a). However, this increases still further the number of contacting surfaces. Liquid-flow cooling is sometimes needed for multi-element sandwiches used in macrosound applications [10]. Most modern sonar transducers are multi-element asymmetric sandwiches with the output end-section tapered-out to achieve the required bandwidth [17]. The basic structure of such a transducer is sketched in Fig. 8b, reproduced from Reference 17. Mechanically, these sonar transducers are very complex structures. They not only employ many stacked-up ceramic rings, with head and tail pieces and stress bolt, but also other elastic elements, serving as washers and spacers, all of which must be included in the mechanical circuit. The final equivalent circuit is usually too complex to be dealt with manually, and computer analysis is essential. There is no space here to enter into detail, and the reader is referred to Refences 17 and 18.

The frequency equation for a multi-element sandwich can be derived in the same way as before, and the result for an n-element $\lambda/4$ section appears as a natural extension of Equations 1 and 5 which refer to n = 2 and n = 3 respectively.

$$\sum_{r} \frac{Z_r}{Z_s} \tan \theta_r \tan \theta_s = 1$$
 (6)

where the summation extends over all of the elements taken two at a time, so that there are nC_2 terms in the sum. The velocity and strain distributions, and from these the transformation ratio, effective coupling and equivalent mass, can be obtained by tracing a plane wave through the system.

Practical considerations

The ceramic material

For emitter applications, the ceramic should have a high Curie Point, high coupling, reasonably low dielectric loss at high-drive; and stable properties, not too dependent on time and temperature. Several manufacturers supply lead titanate zirconate materials with a good compromise of desirable properties [14].

The material is usually in the form of circular plates with fired-on or plated-on silver electrodes. The end-masses make direct contact with the metal plating and the central live lead is taken from a thin sheet, usually of copper, inserted between the discs. The diameter of the ceramic plates is made $< \lambda/4$ extensional-mode so as to avoid any possibility of lateral coupling which might polarize the motion in some direction other than axial, or, worse, excite an unwanted mode. The total disc thickness should be $\ge \lambda/10$ so as to retain a reasonably high coupling and power-handling capacity.

The properties of the ceramic are modified under compression-bias. In particular, the maximum safe temperature at which the material will function efficiently may be drastically reduced. The reader is referred to manufacturers' data for further information [14].

The metal end-sections

As already mentioned, the choice of metals affects the transforming properties of the sandwich. Aside from this, it is often good design to make the back end-section and prestressing bolt of the same material, usually steel. In this way, strains due to differential thermal expansion are avoided if the transducer is driven hard enough to generate appreciable heat. Such strains would change the stress-bias and perhaps weaken the structure.

Internal losses in the metals are always small enough to be ignored, as the material Q-values are always \gg the final system Q_m .

The contacting surfaces

Good mechanical coupling is essential between the various sections of the sandwich if the system is to vibrate as a whole. Some manufacturers use adhesive bonds between the crystal plates and end-sections. In this case, an electrically-conducting adhesive, for example silver-loaded epoxy resin, should be used. This is not very satisfactory for power transducers, as the adhesive bond will fatigue and mechanical coupling may be lost. Other manufacturers insert films of a malleable but non-oxidizable metal which will flow under compression to fill up any crevices and correct any lack of parallelism [8]. The best method is probably to assemble the pieces dry and without inserts. In this case, all mating surfaces must be machined very flat and perpendicular to the axis and the faces of the crystal plates must be parallel to close tolerances. All pieces are ultrasonically cleaned before assembly.

The pre-compression bolt

The most important property of the bolt-material is a very

high tensile strength. It must not only supply a compressive stress of the order of 3 kg/mm^2 over the transducer surface, but it is desirable that the sectional area of the bolt should be as small as possible, that is, its compliance should be high. If T_0 is the required compressive stress and A_C the area of the ceramic, then the bolt must generate a force $F_0 = T_0 A_C$. F_0 is opposed mainly by frictional forces in the screwthreads. The torque needed to tighten the bolt to the required extent is $M_0 = R\mu F_0 = R\mu T_0 A_C$ where R is the radius of the bolt and μ is the coefficient of static friction between the contacting surfaces. μ may range between about 0.10 and 0.17 and so the torque required can be calculated. Dynamic frictional losses in the screw-threads account for most of the mechanical damping in the system, so that well-fitting threads are called for.

References

- Langevin, P. French Patent Nos 502913 (29.5.1920); 505703 (5.8.1920); 575435 (30.7.1924).
- 2 Jaffe, B. et al. Journal of Applied Physics, 25, (1954), pp. 809-810.
- Wright, H.D. and Lane, E.G. Accelerometer, US Patent No. 2714672 (2.8.1955) Priority (30.8.1952).
- 4 Miller, H.B. J. of Acoust. Soc. of America, 35, (1963), p. 1455 (L).
- 5 Miller, H.B. US Patent No. 293912 (1960), Priority (1956).
- 6 Canadian Patent No. 647387.
- 7 Improvements in or relating to electro-mechanical transducer system. British Patent No. 868784 (25.5.1961), Priority (27.8.1956).
- 8 Improvements in or relating to ultrasonic transducers. British Patent No. 957802 (13.5.1964) Priority (12.6.1959).
- 9 Composite electro-mechanical transducer. British Patent No. 902234 (1.8.1962) Priority (13.1.1959).
- 10 Jones, J.B. and Maropis, N. Transducer assembly. US Patent No. 3283182 (1.11.1966) Priority (11.5.1965).
- 11 Mason, W.P. Piezoelectric Crystals and their Application in Ultrasonics. D. van Nostrand Co. Inc. (1950), New York.
- 12 van der Burgt, C.M. and Stuijts, A.L. Developments in ferrite ceramics with strong piezomagnetic coupling. *Ultrasonics*, 1 (1963), pp. 199-210.
- 13 Neppiras, E.A. The effect of shape and internal impedance on the power-handling capacity and efficiency of ultrasonic transducers. Acustica, 15, (1965), pp. 58-62.
- 14 Berlincourt, D. Ultrasonic Transducer Materials. O.E. Mattiat, (Editor), Plenum Press, New York, Chapter 2, (1971).
- 15 Neppiras, E.A. Mechanical transformers for producing very large motion. Acustica, 13, (1963), pp. 368-370.
- 16 Balamuth, L. Mechanical impedance transformers in relation to ultrasonic machining. IRE Transactions on Ultrasonic Engineering, 2, (1954), pp. 23-33.
- 7 Camp, L.W. Underwater Acoustics. John Wiley & Sons Inc., New York (1970).
- 18 Martin, G.E. Vibrations of coaxially-segmented longitudinally-polarized ferroelectric tubes. J. of Acoust. Soc. of America, 36, (1964), pp. 1496-1503.

Table 1 Properties of materials used in constructing sandwich transducers

Material	Young's modulus Y [N/m²]	Density ρ [kg/m²]	Velocity of sound $v = (Y/\rho)^{1/2}$ [m/s]	Characteristic impedance $Z = \rho V = [\rho Y]^{\frac{1}{2}}$ $[kg/m^2s]$	Dynamic fatigue stress F [N/m²]	Thermal expansion [/°C]
	x 10 ¹¹	x 10 ³		x 10 ⁷	x 10 ⁷	x 10 ⁻⁶
Tool steel (KE 672)	2.2	7.8	5200	4.12	55	12
Aluminium bronze (Hidurax special)	1.43	8.5	4070	3.46	37	
Naval brass (BS 251)	0.89	8.45	3240	2.74	15	20
Titanium alloy (IMI 318A)	1.06	4.42	4900	2.17	72	9
Duralumin (DTD 363)	0.74	2.8	5130	1.43	19	23
Ceramic PZT 4 PXE 4	0.77*	7.5 7.45	3220†	2.40‡	≈ 4.0 ≈ 2.5	1-4 1-4
Epoxy cement (hot- set)					≈4.0	
Epoxy cement (cold- set)					≈2.5	

 k_{33} for PZT 4 and PXE 4 approx. 0.70

 $Q_{\epsilon}^{\mathbf{S}}$ for PZT 4 and PXE 4 approx. 200—400 at low drive

[•] $y_{33}^D + v_3^D \ddagger z_{33}^D$